Testing Gravity Using Type Ia Supernovae Discovered by Next-Generation Wide-Field Imaging Surveys

Thematic Areas:
- □ Planetary Systems
- ■ Cosmology and Fundamental Physics
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Abstract: In the upcoming decade cadenced wide-field imaging surveys will increase the number of identified $z < 0.3$ Type Ia supernovae (SNe Ia) from the hundreds to the hundreds of thousands. The increase in the number density and solid-angle coverage of SNe Ia, in parallel with improvements in the standardization of their absolute magnitudes, now make them competitive probes of the growth of structure and hence of gravity. The peculiar velocity power spectrum is sensitive to the growth index $\gamma$, which captures the effect of gravity on the linear growth of structure through the relation $f = \Omega_M^\gamma$. We present the first projections for the precision in $\gamma$ for a range of realistic SN peculiar-velocity survey scenarios. In the next decade the peculiar velocities of SNe Ia in the local $z < 0.3$ Universe will provide a measure of $\gamma$ to $\pm 0.01$ precision that can definitively distinguish between General Relativity and leading models of alternative gravity.

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1 Introduction

In the late 1990’s, Type Ia supernovae (SNe Ia) were used as distance probes to measure the homogeneous expansion history of the Universe. The remarkable discovery that the expansion is accelerating has called into question our basic understanding of the gravitational forces within the Universe. Either it is dominated by a “dark energy” that is gravitationally repulsive, or General Relativity is inadequate and needs to be replaced by a modified theory of gravity. It is only appropriate that in the upcoming decade, with their sheer numbers, solid-angle coverage, and improved distance precisions, SNe Ia will provide measurements of the inhomogeneous motions of structures in the Universe that will provide an unmatched test of whether dark energy or modified gravity is responsible for the accelerating expansion of the Universe.

In the next decade, SNe Ia will be used as peculiar-velocity probes to measure the influence of gravity on structure formation within the Universe. Peculiar velocities induce scatter along the redshift axis of the SN Hubble diagram, which is pronounced at low redshifts and when the magnitude scatter (e.g. due to intrinsic magnitude dispersion) is small. The peculiar velocity power spectrum is sensitive to the growth of structure as \( P_{vv} \propto (fD)^2 \), where \( D \) is the spatially-independent “growth factor” in the linear evolution of density perturbations and \( f \equiv \frac{d\ln D}{d\ln a} \) is the linear growth rate where \( a \) is the scale factor \([12, 7]\). The \( \Lambda \)CDM prediction for the \( z = 0 \) peculiar velocity power spectrum is shown in Figure 1. The growth of structure depends on gravity; \([18]\) find that General Relativity, \( f(R) \), and DGP gravity follow the relation \( f \approx \Omega_M^\gamma \) with \( \gamma = 0.55, 0.42, 0.68 \) respectively (see \([15]\) for a review of these models). Using this parameterization to model gravity, peculiar velocity surveys probe \( \gamma \) through \( fD \), whose \( \gamma \)-dependence is plotted in Figure 2 of \([19]\).

Peculiar velocity surveys have already been used to measure \( fD \) (also referred to as \( f\sigma_8 \)),

![Figure 1: Volume-weighted peculiar velocity power spectrum \( k^3P_{vv}(z = 0) \) for \( \mu \equiv \cos (\hat{k} \cdot \hat{r}) = 1, 0.5 \) (magenta, cyan) where \( \hat{r} \) is the line of sight, as predicted for General Relativity in the linear regime. Overplotted are peculiar-velocity power-spectrum shot noise (diagonal lines) for various observing parameters. Red shows the shot noise expected from a 2-year LSST survey while black shows a 10-year LSST survey. The dotted and dashed lines indicate the assumed intrinsic magnitude dispersion, using 0.08 (dashed) or 0.15 mag (dotted). The expected shot noise from TAIPAN is shown in green (dash-dotted). The bottom solid grey horizontal lines show the approximate range of \( k \) expected to be used in surveys with corresponding redshift depths \( z_{\text{max}} \).](attachment:image)
though not to a level where gravity models can be precisely distinguished. [2] use 6dFGS peculiar velocities using Fundamental Plane distances of elliptical galaxies to estimate absolute magnitudes with \( \sim 0.43 \) mag precision, yielding a 15% uncertainty in \( fD \) at \( z \approx 0 \). The upcoming TAIPAN survey [6] will obtain Fundamental Plane galaxies with densities of \( n_g \sim 10^{-3} h^3 \text{Mpc}^{-3} \), and the WALLABY+WNSHS surveys [17] will obtain Tully-Fisher distances (based on the \( \sim 0.48 \) mag calibration of absolute magnitude based on the HI 21cm line width) of galaxies with densities \( n_g \sim 2 \times 10^{-2} - 10^{-4} h^3 \text{Mpc}^{-3} \) from \( z = 0 - 0.1 \) covering 75% of the sky. These surveys combined are projected to have 3% uncertainties in \( fD \) [11]. For reference, DESI projects a 10% precision of \( fD \) at \( z \approx 0.3 \) by looking for signatures (Redshift Space Distortions; RSD) expected from galaxies infalling toward mass overdensities. Relative to galaxies with Fundamental Plane or Tully-Fisher distances, SN Ia host galaxies currently have significantly lower number density but have better per-object peculiar velocity precision. Existing SN Ia samples have been used to test and ultimately find spatial correlations in peculiar velocities that may be attributed to the growth of structure [9, 1, 16, 13, 14]. SNe Ia discovered by ASAS-SN, ATLAS, and ZTF [20, 21, 4] over the next several years will provide first probative measures of \( fD \) at \( z < 0.1 \).

Two advances in the upcoming decade will make SN Ia peculiar velocities more powerful. First, the precision of SN Ia distances can be improved. The commonly-used empirical 2-parameter spectral model yields absolute magnitude dispersion \( \sigma_M \gtrsim 0.12 \) mag. However, SNe transmit more information than just the light-curve shape and single color used in current SN models. Recent studies indicate that with the right data, SN absolute magnitudes can be calibrated to \( \sigma_M \lesssim 0.08 \) mag [see e.g. 3, 8]. Though not yet established, it is anticipated that such a reduction in intrinsic dispersion comes with a reduction in the magnitude bias correlated with host-galaxy properties that is observed using current calibrations. At this precision the intrinsic velocity dispersion at \( z = 0.028 \) is \( 300 \text{ km s}^{-1} \), i.e. a single SN Ia is of such quality as to measure a peculiar velocity with \( S/N \sim 1 \). If corrections of all SNe Ia are not possible, the use of SN Ia subclasses is an option though at the expense of reducing the numbers of velocity probes. Secondly, in the upcoming decade cadenced wide-field imaging surveys such as ZTF and LSST will increase the number of identified \( z < 0.3 \) Type Ia supernovae from the hundreds to the hundreds of thousands; over the course of 10-years, LSST will find \( \sim 150,000 \) \( z < 0.2 \), \( \sim 520,000 \) \( z < 0.3 \) SNe Ia for which good light curves can be measured, corresponding to a number density of \( n \sim 5 \times 10^{-4} h^3 \text{Mpc}^{-3} \). This sample has comparable number density and more galaxies at deeper redshifts than projected by WALLABY and TAIPAN. With similar densities, the (two) ten-year SN Ia survey will have a (6) \( 29\times \) reduction in shot-noise, \( \sigma^2_M/n \), relative to the Fundamental Plane survey of TAIPAN.

Given these advances, supernovae discovered by wide-field searches in the next decade will be able to tightly constrain the growth of structure in the low-redshift Universe. For example, over the course of a decade a SN survey relying on LSST discoveries plus spectroscopic redshifts can produce 4–14% uncertainties in \( fD \) in 0.05 redshift bins from \( z = 0 \) to 0.3, cumulatively giving 2.2% uncertainty on \( fD \) within this interval, where at \( 0 < z < 0.2 \) most of the probative power comes from peculiar velocities and at higher redshifts from RSD [10].

2 Testing Gravity with Peculiar Velocity Surveys

While the growth rate \( fD \) can be used to test several aspects of physics beyond the standard cosmological model (e.g. dark matter clustering, dark energy evolution), our scientific interest is
in probing gravity, so here we focus on the growth index $\gamma$. To illustrate the distinction, 
$$\frac{d(ln fD)}{d\gamma} = \ln \Omega_M + \int \Omega_M^1 \ln \Omega_M d \ln a \approx -1.68, -0.75, -0.37 \text{ at } z = 0, 0.5, 1.0 \text{ respectively in $\Lambda$CDM; two surveys with the same fractional precision in } fD \text{ will have different precision in } \gamma, \text{ with the one at lower redshift providing the tighter constraint.}$$

In this section, we demonstrate that peculiar velocity surveys in the upcoming decade can measure $\gamma$ precisely for a range of survey-parameter choices.

We project uncertainties on the growth index, $\sigma_\gamma$, for a suite of idealized surveys using a Fisher matrix analysis similar to that of [10, 11] (there is an alternative approach using an estimator for the mean pairwise velocity [5]). The “cross-correlation” analysis incorporates both galaxy overdensities and peculiar velocities. The Fisher information matrix is

$$ F_{ij} = \frac{\Omega}{8\pi^2} \int_{r_{\text{min}}}^{r_{\text{max}}} \int_{k_{\text{min}}}^{k_{\text{max}}} \int_{-1}^{1} r^2 k^2 \text{Tr} \left[ C^{-1} \frac{\partial C}{\partial \lambda_i} C^{-1} \frac{\partial C}{\partial \lambda_j} \right] \, d\mu \, dk \, dr $$ (1)

where

$$ C(k, \mu) = \begin{bmatrix} P_{\delta\delta}(k, \mu) + \frac{1}{n} & P_{v\delta}(k, \mu) \\ P_{v\delta}(k, \mu) & P_{vv}(k, \mu) + \sigma_v^2 \end{bmatrix} $$ (2)

and the parameters considered are $\lambda \in \{\gamma, bD, \Omega_M\}$. The parameter dependence enters through $fD$ in the relations $P_{vv} \propto (fD^2)^2$, the SN Ia host-galaxy count overdensity power spectrum $P_{\delta\delta} \propto (bD + fD^2)^2$, and the galaxy-velocity cross-correlation $P_{v\delta} \propto (bD + fD^2) fD$, where $b$ is the galaxy bias and $\mu \equiv \cos(k \cdot \hat{r})$ where $\hat{r}$ is the direction of the line of sight. While the $bD$ term does contain information on $\gamma$, its constraining power is not used here. Both $f$ and $D$ depend on $\Omega_M = \frac{\Omega_{M0}}{\Omega_{M0} + (1-\Omega_{M0})a^3}$. The uncertainty in $\gamma$ is $\sigma_\gamma = \sqrt{(F^{-1})_{\gamma\gamma}}$. Non-GR models may also predict a change in the scale-dependence of the growth or non-constant $\gamma$, such observations provide additional leverage in probing gravity but are not considered here.

The uncertainty $\sigma_\gamma$ of a survey depends on its solid angle $\Omega$, depth given by the comoving distance out to the maximum redshift $r_{\text{max}} = r(z_{\text{max}})$, duration $t$ through $n = \epsilon \phi t$ where $\phi$ is the observer-frame SN Ia rate and $\epsilon$ is the sample-selection efficiency, and the intrinsic SN Ia magnitude dispersion through the resulting peculiar velocity intrinsic dispersion $\sigma \approx (\frac{5}{ln 10})^{1+z} \frac{1+z}{5} \sigma_M$.

We consider SN peculiar velocity surveys for a range of redshift depths $z_{\text{max}}$ for durations of $t = 2$ and 10 years. The other survey parameters $\Omega = 3\pi, \epsilon = 0.65, \sigma_M = 0.08$ mag are fixed. The $k$-limits are taken to be $k_{\text{min}} = \pi/r_{\text{max}}$ and $k_{\text{max}} = 0.1 \, h \text{Mpc}^{-1}$. A minimum distance $r_{\text{min}} = r(z = 0.01)$ is imposed as our analysis assumes that peculiar velocities are significantly smaller than the cosmological redshift. The sample-selection efficiency $\epsilon$ is redshift-independent, i.e. the native redshift distribution is not sculpted. The input bias of SN Ia host galaxies is set as $b = 1.2$. An independent measurement of $\Omega_{M0} = 0.3 \pm 0.005$ is included and is a non-trivial contributor to the $\gamma$ constraint. Number densities are taken to be direction-independent, neglecting the slight declination-dependence of SN-survey time windows.

All the surveys considered provide meaningful tests of gravity. The projected uncertainty in $\gamma$ achieved by the suite of surveys are shown in Figure 2. The primary result is for the cross-correlation analysis that uses overdensities (RSD), peculiar velocities, and their cross-correlations. The short and shallow, 2-year, $z_{\text{max}} = 0.11$ survey has $\sigma_\gamma \approx 0.038$, which can distinguish between General Relativity, $f(R)$, and DGP gravities at the $>3\sigma$ level. The 10-year survey performance asymptotes at $z_{\text{max}} \sim 0.2$ at a precision of $\sigma_\gamma \sim 0.01$. Figure 2 also shows uncertainties based on
Figure 2: The projected uncertainty in $\gamma, \sigma_\gamma$, achieved by two-year (red) and ten-year (black) SN Ia surveys of varying depth $z_{\text{max}}$. For each survey uncertainties are based on three types of analyses: using only peculiar velocities (dashed); using both RSD and peculiar velocities independently (dotted); using both RSD, peculiar velocities, and their cross-correlation (solid).

two other analyses, one that only uses peculiar velocities, and one that combines independent RSD and peculiar velocity results. Peculiar velocities alone account for much of the probative power of the surveys. RSD alone do not provide significant constraints. However, considering RSD and velocity cross-correlations decreases $\sigma_\gamma$ by $\sim 20\%$. The implication is that there are important $k$-modes that are sample variance limited either in overdensity and/or peculiar velocity who benefit from the sample-noise suppression engendered by cross-correlations.

Survey performance is examined in more detail by considering how $\sigma_\gamma$ in the cross-correlation analysis changes with respect to the survey parameters $\Omega$, $z_{\text{max}}$, $t$, and $\sigma_M$, and also with respect to differential redshift bins within a given survey. Though not directly a survey parameter, we also examine changes with respect to our fiducial choice of $k_{\text{max}}$.

**Solid Angle $\Omega$:** The Fisher Matrix $F$ is proportional to the survey solid angle $\Omega$ so $\sigma_\gamma \propto \Omega^{-1/2}$.

**Differential Redshift Bin $z$:** Certain redshifts constrain $\gamma$ more strongly than others. If at a given moment of a survey we had a set of SNe Ia from which to choose, it turns out the one with the lowest redshift would be preferred. This is demonstrated to be the case at the end of both 2- and 10-year surveys with $z_{\text{max}} = 0.2$. The left panel of Figure 3 shows $|\partial \sigma_\gamma / \partial z|$, which for both surveys monotonically decreases from $z = 0.01$ out to $z = 0.2$. If we had to sculpt the distribution, the preference would be to cut out the highest redshift bins resulting in a decreased $z_{\text{max}}$. The optimal redshift distribution is thus the unsculpted SN-discovery distribution truncated by $z_{\text{max}}$.

**Redshift Depth $z_{\text{max}}$:** Increasing the survey redshift depth increases the $\gamma$ precision. The differential improvement in $\sigma_\gamma$ plateaus at $z_{\text{max}} \sim 0.2$ as seen in Figure 2.

**Survey duration $t$: Intrinsic Magnitude Dispersion $\sigma_M$:** An increased survey duration accumulates more supernovae, decreasing shot noise and increasing the precision in $\gamma$ for all the surveys considered. The surveys we consider have varying relative contributions of sample variance and shot noise: those that have a larger shot-noise contribution (i.e. shorter surveys and those with higher $z_{\text{max}}$) benefit more from extending the survey duration. Like survey duration, intrinsic magnitude dispersion is related to survey performance through the shot noise and thus has a similar relation-
Figure 3: Left: $\sigma_\gamma^{-1}\partial\sigma_\gamma/\partial z$ after two and ten years for a survey with limiting depth $z_{\text{max}} = 0.2$. Right: $\sigma_\gamma^{-1}\partial\sigma_\gamma/\partial \ln t$ (red) and $\sigma_\gamma^{-1}\partial\sigma_\gamma/\partial \sigma_M$ (black) each as a function of $z_{\text{max}}$ for two- (dashed) and ten-year (solid) surveys.

With $\sigma_\gamma$; the effect of duration and magnitude dispersion are shown in the right-panel plot of Figure 3 as $\sigma_\gamma^{-1}\partial\sigma_\gamma/\partial \ln t$ and $\sigma_\gamma^{-1}\partial\sigma_\gamma/\partial \sigma_M$ as a function of $z_{\text{max}}$ for two- and ten-year surveys.

Minimum length scale, maximum wavenumber $k_{\text{max}}$: There is a minimum length scale at which density and velocity distributions are reliably predicted from theory. Changes in this scale engender fractional changes in the $\gamma$ precision as $\sigma_\gamma^{-1}\partial\sigma_\gamma/\partial k_{\text{max}} = 0.0050$ at $k_{\text{max}} = 0.1 h\, \text{Mpc}^{-1}$, which is survey-independent.

3 Conclusions

In the next decade, the high number of SN discoveries together with improved precision in their distance precisions will make $z < 0.3$ SNe Ia, more so than galaxies, powerful probes of gravity through their effect on the growth of structure. Different survey strategies can be adopted to take advantage of these supernovae, and in this White Paper we present a formalism and code (available at [http://tiny.cc/PVScience](http://tiny.cc/PVScience)) by which their scientific merits can be assessed and present results for a range of options.

No other probe of growth of structure or tracer of peculiar velocity can alone provide comparable precision on $\gamma$ in the next decade. At low redshift, the RSD measurement is quickly sample variance limited (as are the planned DESI BGS and 4MOST surveys) making peculiar velocities the only precision probe of $fD$. TAIPAN and a TAIPAN-like DESI BGS will be able to measure FP distances for nearly all usable nearby galaxies, so at low-$z$ the Fundamental Plane peculiar-velocity technique will saturate at a level that is not competitive with a 2-year SN survey.

Combined low-redshift peculiar velocity and high-redshift RSD $fD$ measurements are highly complementary as together they probe the $\gamma$-dependent shape of $fD(z)$ (not just its normalization) and potential scale-dependent influence of gravitational models, since low- and high-redshift surveys are weighted by lower and higher $k$-modes respectively. SN Ia peculiar velocity surveys are of the highest scientific interest and we encourage the community to develop aggressive surveys in the pursuit of testing General Relativity and probing gravity.
References


